CENTER OF PLANNING AND ECONOMIC RESEARCH

LECTURE SERIES

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By
ROBERT EISNER
Northwestern University

ATHENS, 1966

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CENTER OF PLANNING AND ECONOMIC RESEARCH

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GEORGE COUTSOUMARIS

Director General

SOME FACTORS IN GROWTH RECONSIDERED

High levels of employment, investment and output, long-lived capital and growth! How compatible are all of these with each other? What are their interrelations?

The two lectures* which follow share a common approach. They essay to apply relatively simple abstract models to illuminate certain paradoxes of both theoretical and policy significance. In both lectures we endeavour to challenge some aspects of newly conventional wisdom or, at least, to expose a few contradictions in accepted tenets.

It may be well to set the stage by recalling the origin of Harrod-Domar growth models in their context of real or imagined Keynesian unemployment. However desirable (or undesirable) growth might be for its own sake, a central issue posed in the thirties and forties was the attainment and maintenance of high levels of employment. This was presented as the problem of maintaining a rate of investment demand sufficient to absorb full employment saving. And whatever else contributed

* Delivered at the Center of Planning and Economic Research in 1966.

to investment demand, it was suggested by Harrod and by Domar that the maintenance of a sufficient rate of investment demand depended essentially on the maintenance of a sufficient rate of growth of the effective demand for output. Without that growth in the demand for output, adequate investment would not appear profitable and would not, therefore, be sustained.

The Harrod-Domar formulations relating growth, investment and employment were challenged, as were the original Keynesian formulations, for ignoring potential processes of adjustment which would blur or erase the stark issue which had been raised. The multi-coloured banner of flexible prices, factor substitution and interest elasticities was brought to the fore once more, with little apparent recognition that it had been used before.

I have elsewhere striven to set straight the record of the debate. No one has denied, I have insisted, the *possible* flexibilities that would ease the supposed inexorability of the growth-investment-employment relation. The issue does remain, as it did with Keynes, whether prices, interest, and factor substitution are *likely* to be sufficient and sufficiently rapid in their effects to make a critical difference.

Discussion during the last decade or so has (perhaps again) inverted the relation under consideration. We have rather tended to ask not how growth contributes to investment and employment, but how investment and employment contribute to growth and output. We may thus ask, given the constraints of the Harrod-Domar model, how unemployment affects growth. And then, granting a possible rôle for interest rates and factor substitution, we may follow an old tradition in relating these to capital longevity or durability. We shall hope to offer some novelty — and illumination — on both these matters in the lectures that follow.

ON UNEMPLOYMENT AND GROWTH *

Does a higher level of employment contribute to a more rapid, *sustained* rate of economic growth? It would be nice if it did. Then those of us in favour of full employment for other reasons would have the additional argument that it contributes to the general desideratum of long-run growth.

Unfortunately, a number of economists have argued recently that the positive relation between employment and growth is a transitory phenomenon. In this note, I shall cite their arguments briefly, and then offer a «cheerful» answer to the question raised above. That answer, it will be apparent, is not without interesting policy implications.

To start with an eminent authority in a basic text, Samuelson has written, «... Sophisticated economists realize that, in going during one year from 6 to 3 per cent unemployed, we might add about 3 per cent (or 3/94) extra to the annual growth rate, for example, raising it temporarily from a customary 4 per cent to 7 per cent per annum. However, they know that such high rates are by nature temporary. Re-attainment of full employ-

* I am indebted to Harry Johnson for comments on a draught of this lecture, however little he may have agreed with it.

ment is a once-and-for-all condition. In the next year we would again find ourselves growing at the previous 4 per cent rate, since there is no further supply of available labour to bring into use».[1]

In a similar vein, A. W. Phillips declares, «It is... true that while unemployment was actually increasing, let us say from 1½ per cent to 2 per cent, output would be rising less rapidly than it would have been if unemployment had been kept at 1½ per cent. But the argument is often phrased as if the steady rate of growth of the economy with unemployment constant at 2 per cent would be less than the steady rate of growth with unemployment constant at 1½ per cent. I doubt whether this is true».[2]

Finally, Harry Johnson argues, «Contrary to the belief of the Commission [on Money and Credit] and many economists, there is no *a priori* reason for expecting a higher normal level of employment... to produce a higher rate of growth. In analyzing this problem it is necessary to recall that the rate of growth measures the proportionate and not the absolute annual increment of output, and that one should exclude the transitory effects of changes in the percentage of unemployment. Put

1. Paul A. Samuelson, *Economics*, fifth edition (McGraw Hill, 1961), p. 806. The wording is not precisely the same, however, in a corresponding passage on page 784 of the sixth edition, (1964).

2. «Employment, Inflation and Growth», *Economica*, February 1962, p. 13.

very crudely in terms of the Harrod-Domar equation, the problem is whether a higher level of employment and income will raise the average proportion of income saved, lower the marginal capital-output ratio, or (more realistically) raise the savings ratio sufficiently to offset a raised marginal capital-output ratio».[1]

In view of the specific analytical context suggested by Johnson, as well as the ready support of his position offered by Stein[2] and Conard,[3] I should like to meet the issue on Johnson's terms. Drawing on a formulation of my own of a decade ago,[4] I shall use a variant of the Domar equation to demonstrate that economic growth *is* reduced by unemployment. More precisely, we shall see that the long-run or «equilibrium» rate of growth, under appropriate assumptions, is negatively related to the long-run or «equilibrium» percentage of the labour force unemployed.

Johnson's argument is deceptive. As can be seen from the quotation above, he concedes that a higher per cent of unemployment would imply a low-

1. «Objectives, Monetary Standards and Potentialities», *The Review of Economics and Statistics,* Supplement: February 1963, p. 141.

2. Herbert Stein, «Comment», *Ibid.*, p. 150.

3. Joseph Conard, «Comment», *Ibid.*, p. 153.

4. «Underemployment Equilibrium Rates of Growth», *American Economic Review,* March 1952, pp. 43-58, reprinted in *Employment, Growth and Price Levels, Hearings before the Joint Economic Committee of the Congress of the United States, Part IV,* (U.S.G.P.O., 38563, 1959) pp. 811-827.

er *absolute* increase in output. But since, with lower employment, output would also be lower, the proportionate or percentage rate of growth would presumably be unaffected. The implicit algebra is indeed impeccable. For writing Y for output and income, and $S = \Delta K$ for saving and investment, we can note that Johnson has asserted that the proportionate rate of growth of output,

$$\frac{\Delta Y}{Y} = \frac{S}{Y} \cdot \frac{\Delta Y}{\Delta K},$$

cannot be increased unless we either increase $\frac{S}{Y}$, raise $\frac{\Delta Y}{\Delta K}$ (lower the marginal capital-output ratio, $\frac{\Delta K}{\Delta Y}$), or raise $\frac{S}{Y}$ more than we lower $\frac{\Delta Y}{\Delta K}$.

The deception lies in holding the marginal capital-output ratio invariant with respect to the percentage of unemployment. In the Domar model, however, the critical constant is σ, the ratio of the change in *productive capacity* which takes place in an economy in any period to the change in capital stock (investment) which occurs during the same period. It is hence the marginal capital-*capacity* ratio and not the marginal capital-output ratio which is presumed fixed. And this is, after all, as it should be. Investment adds to capacity some amount determined by the production function and factor proportions, which, for certain purposes, we can reasonably take to be given. What happens to output, and along with it the capital-output ratio, is exactly what we have to determine.

In fact, if I may be forgiven for resorting again to some algebra from my fad of the fifties, we may demonstrate the relation between the per cent of employment and the per cent rate of growth by a brief extension of the Domar model. Denoting productive capacity by P and recalling that σ is defined as $\frac{\Delta P}{\Delta K}$, we may write

$$\text{(1)} \qquad \sigma S = \Delta P.$$

Then, denoting the proportion of income saved by

$$\text{(2)} \qquad S = \alpha Y$$

and substituting,

$$\text{(3)} \qquad \Delta P = \alpha\sigma Y.$$

Now if capacity, P, is taken to be the output produced at full employment, the maintenance of full employment, $Y = P$, implies that $\Delta P = \Delta Y$. Hence, substituting in (3) we obtain

$$\text{(4)} \qquad \Delta Y = \alpha\sigma Y,$$

whence

$$\text{(5)} \qquad \frac{\Delta Y}{Y} = \alpha\sigma,$$

the full employment rate of growth.

But let us assume that we have not full employment but some positive proportion of the labour force unemployed. There is every reason to believe that the percentage of productive capacity utilised decreases at least *pari passu* with decreases in the per cent of the labour force employed. This may be recognized in terms of *a priori* considerations about the nature of the production function or merely the empirical and historical observations that lower employment involves less use of physical plant. Recent (1964) figures indicate that with unemployment having remained for a number of years in excess of 5 per cent of the labour force, the corresponding proportion of «idle plant» is in the neighbourhood of 12 or 13 per cent. In the long run we should expect the ratio of output to productive capacity to stabilize and (recognizing, as we did implicitly in our definition of P, that at full employment there may be, under some definitions, idle plant capacity which is not idle in a meaningful aggregative sense) we may write

$$Y = \Theta P. \tag{6}$$

Here we can understand Θ as both the «coefficient of utilisation»,[1] or the per cent of productive capa-

1. This term is due to Domar, «Capital Expansion, Rate of Growth, and Employment», *Econometrica*, April 1946, as reprinted in *Essays in the Theory of Economic Growth* (New York, Oxford Press, 1957), p. 77. For one of the few explorations of underemployment implications of Domar's model, see the section entitled «Less

city which is utilised, and the per cent of the labour force which is employed.

Then, if we start from a position of equilibrium such that

$$Y_0 = \Theta P_0, \tag{7}$$

maintenance of equilibrium requires that

$$\Delta Y = \Theta \Delta P \tag{8}$$

or

$$\Delta P = \frac{\Delta Y}{\Theta} \cdot \tag{9}$$

But

$$\Delta P = \alpha\sigma Y. \tag{3}$$

Hence, substituting (3) in (9), and rearranging terms,

$$\frac{\Delta Y}{Y} = \alpha\sigma\Theta. \tag{10}$$

The equilibrium or long-run rate of growth, if equilibrium or normal employment is less than

Than Full Utilisation of Capacity», pp. 410 - 411, in Ragnar Frisch, «A Reconsideration of Domar's Theory of Economic Growth», *Econometrica*, July 1961. Arguing that the *under*employment approach is «more natural», Frisch states, «A value of θ less than 1 might, unfortunately, best describe the situation in the long run in countries with a free (and not inflationary) economy».

It should be noted that Harrod, as well, envisaged specifically the underemployment implications of his model. Indeed in *Towards a Dynamic Economics*, p. 87, Harrod wrote of his «warranted rate of growth», «G_W is the entrepreneurial equilibrium;... in Keynesian fashion it contemplates the possibility of growing 'involuntary' unemployment».

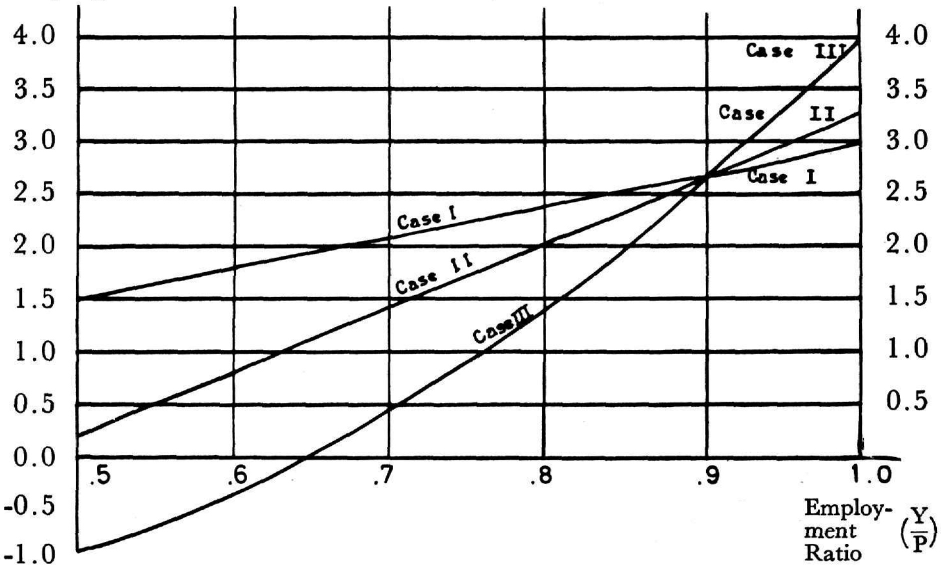
full, is thus $\alpha\sigma\Theta$, where Θ is the proportion of productive capacity utilised or the proportion of the labour force employed. Clearly, the higher the level of employment or, more strictly, the proportion of the labour force employed, the greater is the percentage rate of growth of output.

It is tempting to go further, as some do and as I have done in the past, and note additional implications of this line of analysis if the proportion of income saved is itself dependent upon the level of employment. However, the recent theoretical and empirical work on the consumption function by Friedman and Modigliani, and others working with their models, as well as the earlier findings of Kuznets, raise increasing (although perhaps not definitive) doubts whether saving as a proportion of «permanent» or long-run income is in fact related to either the level of long-run income or the level of employment associated with it. Therefore, abiding by the assumption that saving is a constant proportion of income, we may wish to reconcile the bit of algebra above with Johnson's seemingly plausible argument that, with a constant capital-output ratio, a lower average level of employment implies only a lower average level of output, but the same percentage rate of growth. The rub lies precisely in the distinction between output and capacity which is, after all, at the core of the problem of unemployment. Less than full employment, implying less than full utilisation of capacity, means that each unit of investment adds less to output than it does to capacity. Thus, even with the same proportion of output going to saving and with saving just as productive, in the sense of its effect on productive capacity, the rate of growth will be less to the extent that only a fraction of each year's additional capacity is utilised for additional output. If we were to plot the paths of output and capacity we would, therefore, observe that chronic underemployment not only puts the output curve below the capacity curve but also tilts both curves downward. On the usual semi-log scale employed to show growth over time, we are left, under conditions of underemployment, with parallel straight lines for productive capacity and output but with the slope of these parallel straight lines proportionate to the proportion of the labour force employed.

Since the maintenance of a stable employment ratio or output-to-productive-capacity ratio, Y/P, does imply this equality of the percentage rate of growth of output and the percentage rate of growth of capacity, we can infer the effect of the employment ratio on the rate of growth of output directly from its effect upon the rate of growth of capacity. We can then construct a set of curves relating the employment ratio to the percentage rate of growth under varying assumptions with regard to the effect of the employment ratio on saving and investment. These may include: I, the simple Domar (Johnson) case where the percentage of income saved and invested is constant; II, the case where the percentage of income saved and invested is affected in some simple linear fashion (via the investment function) by the ratio of capacity to output; and III, the case where the percentage of income saved and invested is affected both by the influence of productive capacity on the investment function and the direct influence of unemployment on saving. [This last may reasonably be related to the constraint of provision of at least a minimum subsistence level for all, and to government fiscal policy which will meet this constraint by reducing (disposable income and) saving of the employed without permitting a corresponding increase in saving by the unemployed].

These various cases are portrayed in Figure 1, which shows the growth of productive capacity, $\frac{dP}{P}$, as a function of the employment ratio, $\frac{Y}{P}$. Here the ordinate on any growth-in-productive-capacity curve measures the rate of growth of output and is dependent upon the abscissa, which is the employment ratio. It is to be noted that, in all cases, a higher employment ratio implies a higher percentage rate of growth. This is true, *a fortiori*, where investment is assumed to be reduced in proportion to the excess of productive

FIGURE 1:
The Employment Ratio and the Per Cent Rate of Growth



$\alpha_1 = \cdot 1 \qquad \alpha_2 = \cdot 2 \qquad \alpha_3 = \frac{2}{9} \qquad \beta = \cdot 09 \qquad \sigma = \cdot 3$

Case I: Saving a constant proportion of output

$$S = \alpha_1 Y \qquad \frac{dY}{Y} = \frac{dP}{P} = \alpha_1 \frac{Y}{P} \sigma = \cdot 03 \frac{Y}{P}$$

Case II: Investment demand depressed by excess capacity

$$S = \alpha_2 Y - \beta P$$

$$\frac{dY}{Y} = \frac{dP}{P} = (\alpha_2 \frac{Y}{P} - \beta)\sigma = \cdot 06 \frac{Y}{P} - \cdot 027$$

Case III: Investment demand depressed by excess capacity and saving function depressed by unemployment

$$S = \alpha_3 Y \left(\frac{Y}{P}\right) - \beta P$$

$$\frac{dY}{Y} = \frac{dP}{P} = [\alpha_3 \left(\frac{Y}{P}\right)^2 - \beta]\,\sigma = \frac{1}{15}\left(\frac{Y}{P}\right)^2 - .027$$

In all cases, $\frac{S}{Y} = .1$ when $\frac{Y}{P} = .9$

capacity over output, as shown in Cases II and III. In Case III, which may be defended as depicting at least an intermediate run situation, we find that the rate of growth becomes increasingly sensitive to the employment ratio as that ratio increases. For here we find that because of the form of the assumed effects on both the saving and investment demand functions, the amount saved and invested rises at an increasing rate as the employment ratio rises.

So much for the analytical relations and their graphical presentation! One may, of course, dispute their relevance, which is to dispute the relevance of the Harrod-Domar models. I have endeavoured to defend the general appropriateness of these models elsewhere[1] but that matter is, in any event, not apparently in dispute at this point, at least by Johnson, whose strictures have been my current point of departure.

Whatever the theoretical formulation, one may wish ultimately to fit empirical data. This must be a subtle process, with full and careful specification of the appropriate relations and due attention to lags in the dynamic adjustment process. It should be clear, however, that in terms of a few of the illuminating theoretical tools of the neo-Keynesian era, the reservations of Johnson, Phillips, Samuelson, Stein, Conard, and undoubtedly others are unfounded. Whether or not, with full employment, one should wish a higher rate of growth than is generated by the market, may well be questioned. But the proposition that fuller employment contributes to greater long-run economic growth remains logically sound.

1. «On Growth Models and the Neo-Classical Resurgence», *Economic Journal*, December 1958, pp. 707-721, reprinted in *Employment, Growth and Price Levels, Hearings Before the Joint Economic Committee of the Congress of the United States, Part IV*, (U.S.G.P.O., 38563, 1959), pp. 829-844.

INVESTMENT DURABILITY AND GROWTH*

1. On a visit some years ago to a certain socialist country I was a bit shocked to see what I had taken to be among the more bald of «western propaganda creations», the existence of virtually new apartment buildings and a relatively new hotel which seemed to have been born old. Outer surfaces were decayed, plaster was peeling and what might have been show-case structures seemed well advanced in the progression to slums.

But surely there must be an answer to «western propaganda» — are there not two sides (at least) to everything? — and, not entirely tongue-in-cheek, I should like to use some rather formal but not too «high-brow» analysis to suggest an answer. My answer will not be to deny the existence of dilapidated new buildings in a socialist economy, let alone prove an impossibility theorem in their regard — after all, I do claim to have seen them with my own eyes. Rather, I intend to convince

* I am indebted to Dale Mortensen for comments on part of a draught of this lecture, to Jon Rasmussen and Jon Joyce for assistance in programming computer calculations, and to the (U.S.) National Science Foundation for general financial support.

you that it may be better that way. It may be better to build «crumby» buildings, buildings that crumble fast, but build more of them. It may be better to sacrifice roundaboutness or longevity for more rapid growth.

In a recent carrying of the Word across the Atlantic, Robert Solow declared, «One only has to ask whether rational saving-investment decisions can be independent of the durability of the structures and equipment involved... In all cases when the anwer is no — that is, in all cases — the rate of return is a useful indicator of the choices facing society, while capital-output ratios are not».[1] I propose to focus on tricky and oft-ignored interrelations which one may expect among durability, rates of return, capital-output ratios and economic growth.

The issue may be fairly joined by raising the question implicit in the housing example which I have just cited. Suppose we are running an economy which, for whatever reasons — war destruction, population growth or movement or the increasing demand for better accommodations to which a growing economy may aspire — requires the services of one million additional dwelling units per year. Suppose further that one has the choice of building brick houses or wooden houses,

1. Robert M. Solow, *Capital Theory and the Rate of Return* (Rand McNally, Chicago, 1964), p. 28.

each of which would furnish identical streams of services except that the brick houses would last longer, say 50 years, while the wooden houses would all be blown over in 20 years. And suppose further that the brick houses were twice as expensive. Imagine that dwelling units in the wooden houses would cost $10,000 apiece while those in the brick houses would cost $20,000 apiece. Recognize that this is an economic world and capital is scarce. What should we do?

In fact, now, it would take $10 billion per year to add a million wooden housing units per year and $20 billion per year to add the brick units — for the first 20 years. It is true that, after 20 years, continued acquisition of 1 million new wooden dwelling units would merely suffice to replace the older units being blown over, while continued acquisition of brick houses would increase the existing stock for a period of fifty years. Indeed, in this simple model, the construction of one million wooden dwelling units per year forever would result in an equilibrium stock of 20 million units while the brick construction would bring us eventually to a stock of 50 million units. But whether it pays to construct the brick houses, which will *eventually* give us more accommodations, depends upon some appropriate measure of rates of return and that gets us into the problem of relating output to the stock of capital which produces it.

We can work our way around the problem of evaluating capital and output in the example we have been discussing by stating explicitly merely that the annual flow of services from the brick and wooden houses is identical — it is only the durability or longevity that differ — and that the brick houses are twice as expensive to build. We can then solve for the rate of return of output to capital which would make the fifty-year stream of gross output equivalent to twice a corresponding twenty-year stream. The algebra in this case is quite simple. Letting C = the cost of a wooden house and Y = the value of annual housing services from either a brick or a wooden house, we have:

$$C = \sum_{t=1}^{20} Y(1 + r)^{-t}, \tag{1.1}$$

$$2C = \sum_{t=1}^{50} Y(1 + r)^{-t}, \tag{1.2}$$

and

$$2\left[\frac{1 - (1 + r)^{-20}}{r}\right] = \frac{1 - (1 + r)^{-50}}{r}. \tag{1.3}$$

The value of r satisfying equation (1.3), or at least one economically plausible solution value, is .016 and the ratio of gross output to capital which this implies is approximately .059 for the wooden

(20 year) houses and, of course, half of that or about. .0295 for brick (50 year) houses.

If we can assume a very simple one-factor, linear homogeneous production function, the choice between wooden and brick houses will now depend only upon the rate of time preference. If this is greater than .016 we will build only wooden houses. For the present value of 50 years of (brick) housing services will then be less than twice the present value of 20 years of (wooden) housing services.[1] (Whether it pays to build any houses will depend upon the actual valuation of output of services in relation to the cost of the capital which produces them. In our example, if the value of annual gross output is 5.9 per cent of the cost of wooden houses, at rates of time preference greater than 1.6 per cent *no* houses will be built, wooden or brick).

Now go back to the socialist economy which was our jumping-off point. Recognize that its social rate of return and time discount are high. Presume that the houses which decay rapidly and appear to be born old are the wooden ones. Is it unwise to construct them rather than devote scarce

1. This will be true so long as $2(1+r)^{-20} - (1+r)^{-50} < 1$. More generally, if a_m = the value of annual output per unit of m-type capital and a_n = the value of output of n-type capital, the condition for preferring m-type capital is that

$$\frac{a_m}{a_n} > \frac{1-(1+r)^{-n}}{1-(1+r)^{-m}} \quad .$$

resources to the longer lasting brick dwelling units ?[1]

2. But let us pose the question of durability within the framework of some general growth models. We may first consider a Harrod-Domar type system, in which there is no room for factor-substitution. We can then incorporate a more general Cobb-Douglas-type production function.

We may formalize the Harrod-Domar case by utilizing a «one-factor», constant marginal returns production function, linear and homogeneous in capital inputs of varying durabilities, m and n. Let us write this

$$Y = a_m K_m + a_n K_n. \tag{2.1}$$

On a steady growth path with only m-type capital then,

$$I_{mt} = (1 + g)\, I_{mt-1} \tag{2.2}$$

$$K_{mt} = I_{mt} \sum_{i=1}^{m} (1 + g)^{-i} \tag{2.3}$$

and, with s = the proportion of output going to gross saving,

$$I_{mt} = s\, Y_t, \tag{2.4}$$

1. If the rate of time discount (social rate of return) were 10 per cent, application of the criterion defined in footnote 1, on p. 29, indicates that twenty-year buildings would be preferred over fifty-year buildings with the same annual services if the twenty-year buildings were less expensive by 14.13 per cent or more. Conversely, fifty-year durability would be preferred only if its cost exceeded the cost of twenty-year durability by less than 16.46 per cent.

(2.5) $$Y_t = a_m s Y_t \frac{1-(1+g)^{-m}}{g},$$

whence

(2.6) $$\frac{1-(1+g)^{-m}}{g} = \frac{1}{a_m s}$$

and

(2.7) $$\frac{\partial g}{\partial a_m} > 0, \frac{\partial g}{\partial s} > 0 \text{ and } \frac{\partial g}{\partial m} > 0.$$

The rate of return, r, on a unit of capital of type-m is defined by

(2.8) $$1 = a_m \sum_{i=1}^{m} (1+r)^{-i}$$

or

(2.9) $$\frac{1-(1+r)^{-m}}{r} = \frac{1}{a_m}.$$

Substituting, we then note that g may also be viewed as determined by

(2.10) $$\frac{1-(1+g)^{-m}}{g} = \frac{1}{s} \cdot \frac{1-(1+r)^{-m}}{r},$$

which is particularly easy to solve for g, given s, m, and r, with the aid of old-fashioned tables of «present values of annuities». This enables us to see for a given rate of return r and given $s<1$ (whence $g<r$) that

(2.11) $$\frac{\partial g}{\partial m} > 0,$$

which implies that although a higher value of m must, with constant r, entail a lower value of a_m, the equilibrium growth path, given the rate of return on capital, is steeper the greater the length of life of capital.

Thus, in our simple case of a one-factor, linear homogeneous production function, if all of output is saved the rate of growth equals the rate of return on capital and the capital-output ratio $= \frac{1-(1+g)^{-m}}{g}$. As long as all of output is not saved, g is less than r and, perhaps surprisingly, for given r and s, g then is higher the higher is m, the length of life of capital. This last reflects the fact that, for given rates of gross saving and positive rates of growth, the longer the life of assets the less the proportion of gross acquisitions must go for replacement and the greater, hence, is net saving. To get some crude feeling for the figures, we may note that if we substitute Solow's estimate in the neighborhood of 25 per cent for the rate of return on capital, r, and assume a length of life of capital of 25 years and a gross saving ratio, s, of .2 we get a rate of growth, g, of about 1.8 per cent. If we let the length of life of capital go to 30 years we get an initial decrease in the rate of growth but an ultimate increase to over 2.8 per cent, as is shown in Table 1.

Another way of seeing what is going on, which may be particularly instructive as we proceed, is to note explicitly the ratio of output which is accounted for by *net* investment. If we define net investment, I^*_t, as the change in the stock of existing capital (without regard to remaining services or age distribution of that stock) we see that $I^*_t = I_t - I_{t-m}$, which, under our assumption of a constant rate of growth, g, comes to

$$(2.12) \qquad I^*_t = s[1-(1+g)^{-m}]Y_t.$$

Recalling that a_m is the stream of output to be expected for each of m years from m-type capital, the steady rate of growth to be expected for investment in m-type capital may be written as

$$(2.13) \qquad g = a_m s[1-(1+g)^{-m}].$$

Writing a_m in terms of r, the rate of return, as in (2.9), we find that

$$(2.14) \qquad g = sr\frac{1-(1+g)^{-m}}{1-(1+r)^{-m}},$$

whence one can derive quickly, as by the gross investment route,

$$(2.15) \qquad \frac{1-(1+g)^{-m}}{g} = \frac{1}{s} \cdot \frac{1-(1+r)^{-m}}{r}.$$

And one may also note that $\frac{\partial g}{\partial s} > 0$, $\frac{\partial g}{\partial r} > 0$ and $\frac{\partial g}{\partial m} > 0$, if m may be thought of as a variable, for reasonable values of the parameters.

We may further observe that, fairly generally, when s is less than 1, g is less than r, and for assets of different longevities but promising the same rate of return, the rate of growth will be greater on the longer-lived investment path. One may wonder why, therefore, faced with assets of unequal expected lives but the same rates of return, one should ever pick the shorter-lived assets. Do not the longer-lived assets dominate? And, if so, is this not leading us away from the rationalisation I was suggesting for planning a rapid progression to slums?

In fact, the contradiction is not real. For what we have not yet considered systematically is the path from shorter-lived to longer-lived assets (or vice-versa). Then we do find, as shown in Table 1, that during the period of transition, which is quite lengthy, lengthening of the average life of capital under the assumptions with which we have worked thus far has the effect of lowering gross output. And at an appropriate rate of discount of future output, which may be taken to be the rate of return on capital, investment promising higher output in the more distant future and a higher long-run growth rate in perpetuity but less output in the near future, may be less desirable than investment with a lower expected future output and long-run growth rate but higher output in the near future.

TABLE 1.

Simple Production Function

No Diminishing Returns, Switch from 25-Year to 30-Year Capital.

$$Y_t = a_m K_{mt} + a_n K_{nt}$$

$K_{mt} = s \sum_{i=m}^{1} Y_{t-i}$, where $Y_{t-i} = (1+g_m)^{-(t-i)}$, and $K_{nt} = 0$, for $t<1$;

$K_{mt} = s \sum_{t=m}^{t+1} Y_{t-i}$, for $0<t<m$; $K_{nt} = s \sum_{i=0}^{t-1} Y_i$ for $0<t<n$

$K_{mt} = 0$, for $t>m-1$; $K_{nt} = s \sum_{i=n}^{1} Y_{t-i}$ for $t>n-1$.

For equilibrium paths for m-type and n-type capital, alone:

$$Y_{mt} = a_m K_{mt} = a_m s \sum_{i=m}^{1} Y_{m,t-i} \qquad Y_{nt} = a_n K_{nt} = a_n s \sum_{i=n}^{1} Y_{n,t-i}$$

[and $Y_{mt} = (1+g_m)^{-t}$ for $t<1$], [and $Y_{nt} = (1+g_n)^{-t}$ for $t<1$],

where where

$$g_m = \lim_{t\to\infty} g_{mt}; \quad g_{mt} = \frac{Y_{mt} - Y_{m,t-1}}{Y_{m,t-1}} \qquad g_n = \lim_{t\to\infty} g_{nt}; \quad g_{nt} = \frac{Y_{nt} - Y_{n,t-1}}{Y_{t-1}}$$

$$a_m = \frac{r}{1-(1+r)^{-m}} \qquad a_n = \frac{r}{1-(1+r)^{-n}}$$

$$sa_m = \frac{g_m}{1-(1+g_m)^{-m}}; \qquad sa_n = \frac{g_n}{1-(1+g_n)^{-n}}.$$

$$g_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}}, \quad \lim_{t\to\infty} g_t = g_n, \qquad k_n = \lim_{t\to\infty} \frac{K_{nt}}{Y_t}.$$

$s = .2$, $r = .25$, $m = 25$, $n = 30$, whence:

$a_m = .2509481$, $K_{mo} = 3.984888$ $a_n = .2503099$, $k_n = 3.995408$

and $g_m = .01828$, and $g_n = .02854$.

TABLE 1 (Continued)

(1)	(2)	(3)	(4)	(5)
t	Y_t	$\frac{Y_t}{Y_{mt}}$	$\frac{Y_t}{Y_{nt}}$	g_t
0	1.00000	1.00000	1.00000	.01828
1	1.01815	.99988	.98990	.01815
2	1.03662	.99975	.97989	.01814
3	1.05543	.99961	.96998	.01814
4	1.07457	.99948	.96017	.01814
5	1.09405	.99933	.95045	.01813
10	1.19681	.99856	.90325	.01811
15	1.30906	.99765	.85829	.01808
20	1.43162	.99659	.81544	.01805
24	1.53768	.99562	.78261	.01802
25	1.56537	.99536	.77459	.01801
26	1.64373	1.02643	.79080	.05006
27	1.72602	1.05847	.80734	.05006
28	1.81243	1.09151	.82424	.05006
29	1.90316	1.12558	.84148	.05006
30	1.99844	1.16072	.85909	.05006
31	2.04842	1.16839	.85614	.02501
32	2.10000	1.17631	.85334	.02518
33	2.15323	1.18449	.85070	.02535
34	2.20819	1.19292	.84820	.02552
35	2.26494	1.20162	.84586	.02570
40	2.57826	1.24942	.83648	.02663
45	2.94884	1.30528	.83114	.02763
50	3.38983	1.37058	.83003	.02870
60	4.52526	1.52656	.83626	.02841
80	7.92167	1.86028	.83383	.02864
100	13.90821	2.27365	.83387	.02855
120	24.41928	2.77891	.83392	.02854
140	42.87169	3.39628	.83393	.02854
160	75.26694	4.15076	.83393	.02854
180	132.14135	5.07285	.83393	.02854
200	231.99221	6.19979	.83393	.02854

We may also wish to explore the consequences of a shift from shorter to longer-lived investment with the same rate of return, on the assumption that the rate of growth of gross investment remains unaffected but that the saving ratio is permitted to change. We then find that the transition period begins with a reduction in output as compared with the output accompanying a shorter-lived investment path. This is due to the lesser output per unit of longer-lived capital. But beginning with the m-th year after the shift, the total stock of capital grows relative to its amount on the shorter-lived investment path. This growth continues until the transition is completed, at the end of n years. On the new equilibrium path, output per unit of capital is less and the stock of capital is greater. As long as the rate of return on capital is greater than the rate of growth, output is greater and, perforce, the ratio of output saved (invested) is less. The rate of growth of output, on the assumption that the rate of growth of investment is unchanged is, however, also unchanged. Again, whether the shift, this time, to a higher but parallel output path is desirable, depends upon the rate of time preference and the parameters of the production function which determine just how much increase in output is achieved.

It is perhaps now time to recognize the limitations of some of our restrictive assumptions as well as just how far our model has carried us. The one-input, linear homogenous production function, for one thing, is perhaps not as restrictive as its patent absurdity may suggest. If we can think of other factors, labour for example, in perfectly elastic supply, as W. Arthur Lewis has done in some of his work, our analysis follows through on the simple assumption that the production function remains homogenous of the first degree in all factors. Then generally diminishing returns never come in as long as constant rates of return to capital and a perfectly elastic supply of other factors (labour) keep factor proportions unchanged. With capital and labour growing at the same rate we may merely think of all of our earlier findings as applying to output per unit of labour.

3. Nevertheless, it will be useful to get back into the mainstream of analysis by working with a production function in capital and labour, with the usual diminishing returns to each factor individually and the possibility of different rates of return, as well, from assets of different longevities. Let us indeed assume a Cobb-Douglas-type production function which is homogeneous of the first degree in labour and capital of types m and n. Specifically, we may now write *gross* output as

$$(3.1) \qquad Y = \frac{a_m K_m + a_n K_n}{K_m + K_n} (K_m + K_n)^{\alpha} L^{1-\alpha}$$

so that, for an economy using only m-type capital

$$(3.2) \qquad Y_m = a_m K_m^\alpha L^{1-\alpha},$$

$$(3.3) \qquad \frac{Y_m}{L} = a_m \left(\frac{K_m}{L}\right)^\alpha,$$

and

$$(3.4) \qquad \frac{K_m}{Y_m} = \frac{1}{a_m} \left(\frac{K_m}{L}\right)^{1-\alpha}.$$

If we assume initially a balanced growth path with Y_m, K_m, and L all growing at a rate g, we may now write

$$(3.5) \qquad Y_m = a_m \left[s\, Y_m \left(\frac{1-(1+g)^{-m}}{g} \right) \right]^\alpha L^{1-\alpha},$$

$$(3.6) \qquad \frac{Y_m}{L} = a_m^{\frac{1}{1-\alpha}} \left[s \left(\frac{1-(1+g)^{-m}}{g} \right) \right]^{\frac{\alpha}{1-\alpha}},$$

$$(3.7) \qquad \frac{K_m}{L} = \left[a_m\, s \left(\frac{1-(1+g)^{-m}}{g} \right) \right]^{\frac{1}{1-\alpha}},$$

$$(3.8) \qquad \frac{K_m}{Y_m} = s \left(\frac{1-(1+g)^{-m}}{g} \right)$$

and

$$(3.9) \qquad \frac{1-(1+g)^{-m}}{g} = \frac{1}{a_m s} \left(\frac{K_m}{L}\right)^{1-\alpha}.$$

Given the capital-labour ratio and a_m and α, the parameters of the production function, the capital-output ratio has already been fixed by (3.4); the balanced rate of growth is then determined by the saving ratio in (3.8) or (3.9), and we are still in a Harrod-Domar world, where $\frac{\partial g}{\partial s} > 0$.[1]

Where the capital-labour ratio is not fixed it can be seen from (3.7) that it is positively related to s, as more saving, of course, means more capital, and positively related to m, because if capital lasts longer, other things being equal, there is always more of it around. The capital-labour ratio is negatively related to the rate of balanced growth, with a given gross saving ratio, because a higher rate of growth means a reduced amount of capital available from the saving out of relatively reduced previous outputs. Output per unit of labour is positively related to the capital-labour ratio and to the parameters a_m and α. We cannot conclude, however, that an increased capital-labour ratio due to increased durability of capital which does not increase the rate of return will necessarily increase the output-labour ratio. The result now will depend upon how far a_m must be lowered to maintain r constant in the face of the increase in m.

For our own purposes it will be instructive to assume a balanced rate of growth, g, and note that the value of a_m consistent with it (and the corresponding capital-labour ratio) is then

1. This presumes that the labour supply is such that the quantity of labour can and does always adjust to the quantity of capital.

$$(3.10) \qquad a_m = \frac{\left[\frac{K_m}{L}\right]^{1-\alpha}}{\left[\frac{1-(1+g)^{-m}}{g}\right]}.$$

The expected rate of return on capital is that value of r for which the sum of expected future marginal products of a unit of capital, discounted at $(1 + r)$ per time period, equals unity. Hence for type-m capital

(3.11)

$$\sum_{i=1}^{m} \frac{\partial Y_{m,t+1}}{\partial K_{mt}} (1+r)^{-i} = \alpha a^m \left[\frac{K_m}{L}\right]^{\alpha-1} \left[\frac{1-(1+r)^{-m}}{r}\right] = 1.$$

The rate of return on the balanced growth path for an economy using m-type capital is thus defined by

$$(3.12) \qquad \frac{1-(1+r)^{-m}}{r} = \frac{\left(\frac{K_m}{L}\right)^{1-\alpha}}{\alpha\, a_m}$$

whence, substituting (3.9),

$$(3.13) \qquad \frac{1-(1+r)^{-m}}{r} = \frac{s}{\alpha}\left[\frac{1-(1+g)^{-m}}{g}\right],$$

and

$$(3.14) \qquad \frac{\partial Y_m}{\partial K_m} = \frac{r}{1-(1+r)^{-m}} = \frac{\alpha}{s\left[1-(1+g)^{-m}\right]}.$$

Examining (3.3), (3.12) and (3.13) we note that a faster rate of growth, for given s and α and a_m, must imply a higher rate of return on capital and, for a given a_m, a lower capital-labour ratio and less output per unit of labour.

By way of numerical illustration, for $g = .03$, $m = 20$, $\alpha = \frac{1}{3}$, and $s = .2$, we have $\frac{K_m}{Y_m} = 2.9755$ and $r = .09316$. The arithmetic value of a_m of course depends on our measure of labour. Let us, for convenience, define a unit of labour so that at the capital-output ratio just indicated there are $1 - \alpha$ (or $\frac{2}{3}$) units of labour for each unit of output.[1] We then find that $\frac{K_m}{L} = 4.4632$ and $a_m = .91105$. All of these values, it must be remembered, relate to the equilibrium path for m-type capital.

For n-type capital, the equilibrium path is of course similar in nature. Changing only the parameter $m = 20$ to $n = 25$, we find $\frac{K_n}{Y} = 3.4826$. Choosing a_n so that r would be unchanged for the same $\frac{K}{L}$, $a_n < a_m$ but $\frac{K_n}{L} > \frac{K_m}{L}$; specifically, $a_n = .84924$, $\frac{K_n}{L} = 5.0864$, and $r = .08253$ at this new capital-labour ratio.

1. It should be understood that while labour's share of output under conventional assumption will remain constant at $1\text{-}\alpha$, output per unit of labour as originally defined will change where a_m changes or the capital-labour ratio changes.

The value of a_n sufficient to keep r unchanged with the *given* equilibrium $\frac{K}{L}$ ratio defined for m-type capital above implies in this case a lower ratio of Y to L when the n-type equilibrium $\frac{K}{L}$ ratio is attained. The value of a_n sufficient to maintain $\frac{Y}{L}$ at the same value would give a new equilibrium rate of return still below that for m-type capital.

We find thus that longer-lived capital promising the same rate of return at the same capital-labour ratio, ultimately implies a higher capital-output ratio, a higher capital-labour ratio, a lower marginal product of capital, a lower rate of return on capital and, in this case, a lower output-labour ratio. As illustrated in Table 2, if the same rate of return were available at the initial capital-output and capital-labour ratios, switching to longer-lived assets would entail an initial downward departure from the steady growth path because each unit of longer-lived capital would be productive of less gross output. With saving the same ratio of output, this relative decline in output would in turn be aggravated by the resultant decline in saving and would continue until the capital-labour ratio began to come up after the last of the shorter-lived capital expired. Through the period of declining capital-labour ratios the expected rate of return on capital calculated (myopically) on the assumption that the current capital-labour ratio would persist, would thus be rising. With the increase in capital-labour ratios beginning after the last of the shorter-lived capital has been retired, the capital-labour, capital-output and output-labour ratios would all rise, but the rate of return on capital would decline. Most, but not all, of the approach to a new equilibrium would be reached when retirement of the longer-lived assets begins. On this new equilibrium path, output will still be lower than it would have been on the old path, and one would have sacrificed a particularly significant amount of output at the beginning of the period of transition.

Generalization from numerical examples can be tricky, however, and so it is again. In particular, whether equilibrium output on the new greater-durability path is higher or lower turns out to depend upon the rate of return which we are keeping constant for given capital-labour ratios, and the precise longevities of capital along the two paths. For, making use of (3.12), if n-type capital is to

TABLE 2

Cobb-Douglas-Type Production Function
Switch to More Durable Capital; r Below Critical Rate.

$$Y_t = \frac{a_n K_{mt} + a_n K_{nt}}{K_{mt} + K_{nt}} (K_{mt} + K_{nt})^{\alpha} L_t^{1-\alpha} \qquad t>0$$

$$K_{mt} = s \sum_{i=m}^{1} Y_{t-i},\ K_{nt} = 0,\ Y_t = (1+g)^t, \text{ and } L_t = \frac{2}{3}(1+g)^t \quad t<1$$

$$K_{mt} = s \sum_{i=m}^{t+1} Y_{t-i}, \qquad K_{nt} = s \sum_{i=0}^{t-1} Y_i \qquad 0<t<m, n$$

$$K_{mt} = 0, \qquad K_{nt} = s \sum_{i=n}^{1} Y_{t-i} \qquad t>m-1,\ n-1$$

$$L_t = L_{t-1}(1+g); \qquad k_t = \frac{K_{mt}+K_{nt}}{L_t}; \qquad y_t = \frac{Y_t}{Y_0(1+g)^t};$$

$$g_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}}.$$

$s = .2;\ m = 20;\ n = 25;\ \alpha = \frac{1}{3};\ g = .03;\ g_t = g$ for $t<1$.

Hence: $a_m = .911048$; $a_n = .849244$; $r_t = .09316258$ for $t<1$;

$K_{mo} = 2.97549498$; $k_o = 4.46324247$.

(1) t	(2) k_t	(3) y_t	(4) g_t
0	4.46324	1.00000	.03000
1	4.46324	.99557	.02544
2	4.46324	.99120	.02547
3	4.46195	.98687	.02550

TABLE 2 (Continued)

(1) t	(2) k_t	(3) y_t	(4) g_t
4	4.45943	.98258	.02553
5	4.45571	.97834	.02555
10	4.42093	.95768	.02565
15	4.36277	.93776	.02569
19	4.30223	.92221	.02571
20	4.28539	.91836	.02571
21	4.26791	.92852	.04139
22	4.41108	.93837	.04093
23	4.55305	.94794	.04050
24	4.69375	.95723	.04010
25	4.83313	.96626	.03971
26	4.97117	.96610	.02983
27	4.96870	.96598	.02987
28	4.96688	.96591	.02992
29	4.96569	.96587	.02996
30	4.96510	.96587	.03000
35	4.97099	.96643	.03020
40	4.99055	.96787	.03038
50	5.05480	.97182	.03018
60	5.06926	.97265	.03008
80	5.08310	.97347	.03002
100	5.08575	.97363	.03000
120	5.08626	.97366	.03000
140	5.08637	.97367	.03000
160	5.08639	.97367	.03000
180	5.08639	.97367	.03000
200	5.08639	.97367	.03000
∞	5.08639	.97367	.03000

offer the same rate of return as m-type capital at any given $\frac{K}{L}$ ratio,

$$(3.15) \qquad \frac{a_n}{a_m} = \frac{1-(1+r)^{-m}}{1-(1+r)^{-n}}.$$

But then we have for n-type capital, analogously to (3.6), the expression

$$(3.16) \qquad \frac{Y_n}{L} = a_n^{\frac{1}{1-\alpha}} \left[s \frac{1-(1+g)^{-n}}{g} \right]^{\frac{\alpha}{1-\alpha}}.$$

Dividing (3.16) by (3.6) and using (3.15) to substitute for $\frac{a_n}{a_m}$, the ratio of n-path equilibrium output to m-path equilibrium output may be written:

$$(3.17)\ y = \frac{Y_n}{Y_m} = \left[\frac{1-(1+r)^{-m}}{1-(1+r)^{-n}}\right]^{\frac{1}{1-\alpha}} \left[\frac{1-(1+g)^{-n}}{1-(1+g)^{-m}}\right]^{\frac{\alpha}{1-\alpha}}.$$

(3.18) Since for $m<n$, $r>0$ and $0<\alpha<1$, $\frac{\partial y}{\partial r}>0$,

and since $y = 1$, that is $Y_n = Y_m$, when

$$(3.19) \qquad \frac{1-(1+r)^{-n}}{1-(1+r)^{-m}} = \left[\frac{1-(1+g)^{-n}}{1-(1+g)^{-m}}\right]^{\alpha},$$

it can be seen that the switch to longer-lived capital entails a rise in the equilibrium path of output for values of r above the critical level defined by (3.19) but a fall in that path for values of r below the critical level. For $\alpha = \frac{1}{3}$, m = 20, n = 25 and g = .03, the critical value of r = .1148. The results of Table 2, in which $y<1$, could thus have been predicted from the fact that $r_m = .09316$, which is less than .1148.

Actually, given α, m, and g, the initial value of r, designated above r_m, is determined in our model by the saving ratio, s, because given the saving ratio there is only one value of a_m which will produce enough output to sustain the rate of growth, g. We may illustrate the consequences of a higher rate of return by assuming a lower saving ratio. Table 3 is therefore like Table 2, except that s =.1. Hence $a_m = 1.14785$ and $r_m = .21984$, which is more than the critical rate, .1148. The value of a_n necessary to promise the same rate of return on n-type capital at the m-path capital-labour ratio is then derived from (3.15); $a_n = 1.13417$.

We now note that with a «high» rate of return, above the critical rate, the switch to more durable investment again brings about an initial lowering of the output path, but the new equilibrium is higher. Indeed, substitution in (3.17) would confirm what Table 3 suggests, that the ratio of output from n-type capital to that which would have been forthcoming from m-type capital will approach 1.0626, a long-run equilibrium gain of 6.26 per cent per year.

TABLE 3.

Cobb-Douglas-Type Production Function.
Switch to More Durable Capital; r above Critical Rate.
Formulation identical to Table 2, except s = .1.
Hence, $a_m = 1.14785$; $a_n = 1.13417$; $r_t = .21984233$ for $t < 1$;
$K_{mo} = 1.48774749$; $k_o = 2.2316212$

(1) t	(2) k_t	(3) y_t	(4) g_t
0	2.23162	1.00000	.03000
1	2.23162	.99922	.02920
2	2.23162	.99845	.02921
3	2.23151	.99769	.02921
4	2.23129	.99693	.02922
5	2.23096	.99617	.02922
10	2.22788	.99249	.02925
15	2.22272	.98895	.02927
19	2.21734	.98620	.02929
20	2.21585	.98552	.02929
21	2.21429	.99711	.04211
22	2.29332	1.00835	.04161
23	2.37174	1.01925	.04114
24	2.44950	1.02983	.04069
25	2.52659	1.04011	.04028
26	2.60298	1.04092	.03080
27	2.60908	1.04173	.03080
28	2.61518	1.04254	.03080
29	2.62127	1.04334	.03080
30	2.62735	1.04415	.03079
35	2.65766	1.04813	.03078
40	2.68773	1.05205	.03076
50	2.74197	1.05864	.03033
60	2.75891	1.06063	.03015
80	2.77223	1.06221	.03003
100	2.77481	1.06251	.03001
120	2.77532	1.06257	.03000
140	2.77543	1.06259	.03000
160	2.77544	1.06259	.03000
180	2.77545	1.06259	.03000
200	2.77545	1.06259	.03000
∞	2.77545	1.06259	.03000

The mappings of (3.17) in Table 4 and, for fewer illustrative values, in Chart I, reveal that for any given rate of return and capital-labour ratio, as would be determined by the elasticity of output with respect to capital (α), the ratio of output saved (s), the rate of growth (g) and the initial longevity (m), the value of y has a minimum for some finite value of n. As y is defined - the ratio of equilibrium output with capital of n-years durability to equilibrium output with capital of m-years durability - it is unity when n = m, but whether $\frac{\partial y}{\partial n} > 0$ in the neighbourhood of m depends upon whether m > the value of n for which y is minimal. If $m > n_{min}$, an increase in longevity (n) will increase y, that is, raise the equilibrium path of output. The greater the increase in longevity the more the path will be raised, although an upper bound for the value of y is then readily seen to be

$$(3.20) \qquad \lim_{n \to \infty} y = \left[\frac{1-(1+r)^{-m}}{[1-(1+g)^{-m}]^{\alpha}} \right]^{\frac{1}{1-\alpha}} ,$$

for r>g, which includes the relevant ranges of these parameters.

Where $m < n_{min}$, however, we find that $\frac{\partial y}{\partial n} < 0$ in the neighbourhood of m, that is, increases in longevity will lower the equilibrium path of output. Large increases in longevity, bringing n beyond n_{min},

TABLE 4.

Cobb-Douglas-Type Production Function

Mapping of y and r as functions of g, s, m, and n; $\alpha = \frac{1}{3}$.

$$y = \left[\frac{1-(1+r)^{-m}}{1-(1+r)^{-n}}\right]^{\frac{1}{1-\alpha}} \left[\frac{1-(1+g)^{-n}}{1-(1+g)^{-m}}\right]^{\frac{\alpha}{1-\alpha}} \quad ; \quad \frac{1-(1+r)^{-m}}{r} = \frac{s}{\alpha}\left[\frac{1-(1+g)^{-m}}{g}\right].$$

	g	.01			.03			.05		
	s	.1	.2	.3	.1	.2	.3	.1	.2	.3
m = 10	n \ r	.3319	.1186	.0301	.3745	.1449	.0512	.4186	.1715	.0723
y	1	2.3804	5.1819	8.4131	2.2257	4.7853	7.7344	2.0922	4.4409	7.1443
	5	.9867	1.4095	1.8189	.9678	1.3574	1.7387	.9526	1.3117	1.6676
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	15	1.1311	.9118	.7311	1.1243	.9332	.7617	1.1159	.9509	.7897
	25	1.3982	.9268	.5235	1.3414	.9609	.5848	1.2904	.9851	.6417
	35	1.6139	1.0046	.4416	1.4893	1.0270	.5211	1.3905	1.0376	.5944
	45	1.7879	1.0907	.4016	1.5909	1.0863	.4940	1.4488	1.0760	.5774
	100	2.3626	1.4274	.3635	1.8060	1.2291	.4791	1.5309	1.1357	.5718
	∞	2.9759	1.7979	.4231	1.8549	1.2623	.4871	1.5368	1.1400	.5732
	min*	.9585	.9018	.3623	.9499	.9295	.4766	.9421	.9501	.5701
		(6.825)	(18.242)	(87.438)	(6.449)	(16.983)	(75.188)	(6.108)	(15.921)	(68.000)

* value of n_{min} indicated in parentheses on line below.

g		.01			.03			.05		
s		.1	.2	.3	.1	.2	.3	.1	.2	.3
m = 20	n \ r	.1777	.0672	.0207	.2198	.0932	.0417	.2650	.1198	.0628
	1	3.7709	9.2005	15.8097	3.2452	7.7869	13.3070	2.8434	6.6982	11.3770
	5	1.1721	2.2006	3.3272	1.0790	1.9527	2.9146	1.0114	1.7597	2.5900
	10	.9462	1.3600	1.7711	.9181	1.2682	1.6264	.9023	1.1971	1.5102
	15	.9465	1.1064	1.2555	.9414	1.0734	1.2049	.9414	1.0484	1.1637
	25	1.0692	.9525	.8486	1.0626	.9737	.8816	1.0534	.9882	.9090
y										
	35	1.2105	.9323	.6796	1.1697	.9754	.7556	1.1311	1.0007	.8186
	45	1.3358	.9536	.5899	1.2480	1.0007	.6943	1.1781	1.0223	.7799
	100	1.7635	1.1629	.4481	1.4165	1.1055	.6234	1.2449	1.0704	.7481
	∞	2.2213	1.4615	.4590	1.4548	1.1352	.6242	1.2496	1.0745	.7484
	min*	.9351	.9316	**	.9165	.9690	.6220	.9020	.9880	.7481
		(12.214)	(33.230)	**	(10.837)	(29.047)	(124.5)	(9.672)	(25.805)	(109.000)
m = 30	n \ r	.1254	.0494	.0174	.1685	.0754	.0384	.2162	.1026	.0595
	1	5.0379	12.8293	22.4995	4.0083	10.0104	17.4476	3.3064	8.0728	13.9694
	5	1.3927	2.9255	4.6898	1.1981	2.3997	3.7866	1.0715	2.0346	3.1528
	10	1.0045	1.7124	2.4674	.9277	1.4839	2.0902	.8868	1.3257	1.8204
	15	.9275	1.3270	1.7294	.8964	1.2050	1.5331	.8880	1.1222	1.3906
	25	.9580	1.0537	1.1442	.9585	1.0273	1.1019	.9643	1.0108	1.0708
y										
	35	1.0466	.9699	.8982	1.0390	.9890	.9304	1.0293	.9993	.9541
	45	1.1402	.9483	.7654	1.1043	.9904	.8444	1.0710	1.0090	.9020
	100	1.4956	1.0572	.5381	1.2519	1.0621	.7322	1.1315	1.0467	.8531
	∞	1.8838	1.3155	.5046	1.2858	1.0897	.7261	1.1358	1.0506	.8524
	min*	.9223	.9477	**	.8960	.9869	**	.8789	.9988	**
		(17.236)	(47.148)	**	(14.362)	(38.891)	**	(12.103)	(32.914)	**

* value of n_{min} indicated in parentheses on line below.

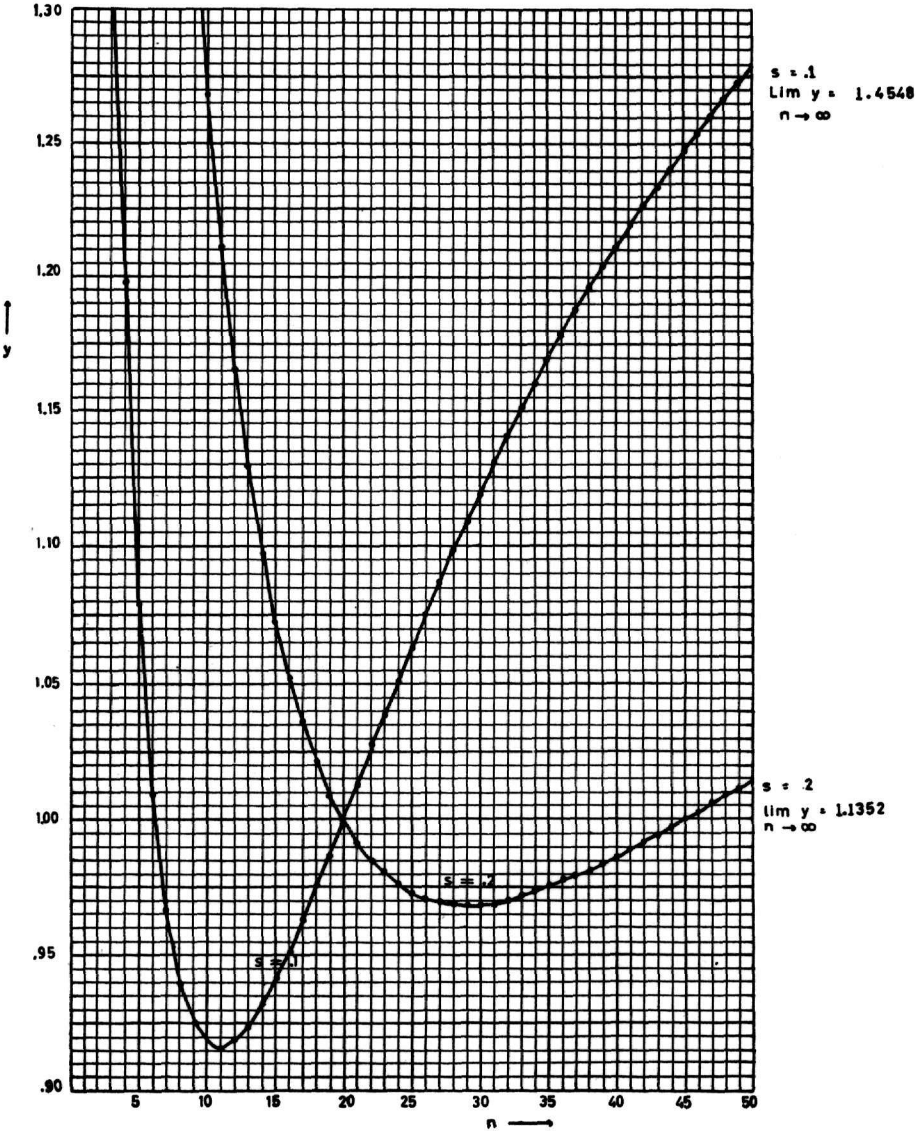
** not calculated; $n_{min} > 128$.

Chart I: Cobb - Douglas - Type Production Function. Mapping of y as a function of n.

$$y = \lim_{t \to \infty} \frac{Y_{nt}}{Y_{mt}} = f(n) \quad \text{for} \quad a = \frac{1}{3}, \; g = .03, \; m = 20:$$

s = .1 (and r = .2198). s = .2 (and r = .0932).

y_{min} = .9165 tor s = .1 ; y_{min} = .9690 tor s = .2.



will turn $\frac{\partial y}{\partial n}$ positive but may or may not raise y above where it was for longevity m. Table 4 makes clear that n_{min} is higher and the limit of y as n approaches infinity is lower for higher values of s. It implies, therefore, that a high rate of saving should militate against moves to greater durability of capital. Chart 1 illustrates much of this by graphing y as a function of n for the values of α, g and m and the two values of s (and hence of r) underlying, respectively, Tables 1 and 2. One may note that for the higher saving ratio (and lower rate of return), the point at which the output path begins to become higher involves considerably greater longevity. Only when the length of life of capital has grown from 20 years to almost 45 years will the same equilibrium path of output be attained; $y = 1.0007$ for $n = 45$. And y will not surpass an upper bound of 1.1352 even as n approaches infinity.

The value of y will be sharply higher for very small values of n. This would suggest that if the production function were such that the rate of return were the same (for unchanged capital-labour ratios) less durable capital would imply a higher equilibrium path of output. Since the resulting lower capital-labour ratio would actually raise the rate of return, one might expect economies to plunge to minimal durability. The fact that they do not would then imply non-optimal qualities in the market mechanism—or the planning system — or, more likely, that the production function is not what we have assumed it to be. There may not, for example, be an unlimited supply of assets of lower durability promising the same rate of return, or our production function may not be Cobb-Douglas, or both.

If one is looking for an intuitive sense of these perhaps bewildering computational and algebraic results, it is this. The increase in longevity, with a constant saving ratio, increases output for a given quantity of labour by a proportion determined by the elasticity of output with respect to capital, α, and the relative increase in capital stock. This last is determined by the relative longevities and the rate of growth. The greater the rate of growth the less the relative increase in capital stock brought about by any particular increase in longevity.

The requirement that the rate of return remain the same for more durable assets, as long as the capital-labour ratio is not changed, forces a reduction in the productivity or scale coefficient; that is, a_n must be less than a_m. Now the proportion that a_n must be less than a_m depends upon the amount of the rate of return which we require be unchanged. If the rate of return is high (and durability is already substantial) an increase in expected longevity of assets, producing a constant stream of annual services, has little effect on the rate of return. Therefore, only a slight reduction in the scale or productivity coefficient is called for; the requirement that r be unaffected will not cause a_n to be less than a_m by an amount sufficient to overbalance the output-increasing effect of the greater capital stock.

As we see things this way we may note that our results will depend critically not merely on the value of α, the constant elasticity of output with respect to capital of the Cobb-Douglas-type production function, but also on the assumption of that form of function to begin with. Thus, even with the somewhat broader class of constant-elasticity-of-substitution production functions, we note that the elasticity of output with respect to capital may (if the elasticity of substitution is less than one) be declining as the capital-labour ratio increases. And, still more generally, we may find that increases in the capital-labour ratio due to increased longevity of assets (or any other cause) may have quite limited output-increasing effects and may produce rapidly declining rates of return. If this is so, almost any reduction in the scale or productivity coefficient necessary to maintain r unchanged at given capital-labour ratios may be sufficient to overbalance this limited output-increasing effect of the higher capital-labour ratio. Or, in terms of our model, the critical level of r may indeed be prohibitively high, so that increases in longevity which offer the same rate of return at given capital-labour ratios can virtually never raise the equilibrium output path, and the essential findings of Table 2 are again relevant.

4. All this may be related to an interesting and perhaps important issue of policy. What is to be said, in the light of our analysis, of the notion that growth may be encouraged by a policy of lower interest rates to stimulate investment coupled with higher taxes to bring about a compensating reduction in demand (in order to avoid inflation). As a limiting case let us imagine that gross saving as a ratio of output is kept constant, as might be accomplished by an *ad valorem* tax on capital goods just sufficient to reduce the rate of return after taxes, given the same rate of saving, by as much as the reduction in the rate of interest.

There is a lot concealed in the last sentence. It is only on capital goods of a single durability that one can talk of a single *ad valorem* tax which will reduce the rate of return by a unique amount. For capital of varying lengths of life we must note that the reduction in rate of return brought about by imposition of a given *ad valorem* tax will be related inversely to the durability of the asset. In the context of the model which we have constructed, we may imagine a tax on capital goods which would make the after-tax rate of return on n-type capital goods, at the capital-labour ratio existing on the *m*-type balanced growth path, correspond to the new lowered rate of interest (or risk-adjusted cost of capital). Such a tax, however, would make m-type capital goods unprofitable, even at this new lower rate of interest. The present value of after-tax returns from m-type capital goods would now be less than their cost, and such a tax-compensated reduction in the rate of interest would induce a switch from m-type to n-type capital goods.[1]

We have already examined the varying characteristics of this switch. If the simple production function of Table 1 is relevant, we know that the rate of growth will in fact be increased — eventually. Whether the increase is desirable depends upon the amount of short-run loss of output and the rate of time preference used to balance the short-run losses and the long-run gains.

The issue is essentially the same if we have the Cobb-Douglas type production function and «high» rate of return on capital of Table 3. For although the interest-tax-induced shift to more durable investment will now not raise the long-run rate of

1. We may note that the rate of return will actually rise during the transition so that, to keep our model internally consistent, we would have to permit further compensating year-to-year tax adustments or, perhaps more realistically, suggest such supply constraints in the capital goods producing industries that rises in supply price, in the face of any attempt to increase the rate of production of capital goods, would choke off demand.

growth, it will still confer long-run benefits — in the form of a higher output path — in exchange for short-run losses.

If the production function and parameters of Table 2 are the relevant ones, however, the shift to more durable investment would not seem warranted. For it would entail less output in the short-run *and* less output in the long-run. More durable capital could only be justified if the rate of return expected from it were sufficiently higher than that expected from less durable capital. But if more durable investment promised a higher rate of return one might expect that it would have been chosen even without the interest-tax-shift inducement.

Indeed, if the parameters of Table 2 apply, one may ask whether we should not rather induce a shift to less durable capital. For this would raise output in both the short-run and long-run. The answer, at least within the frame of reference of our model, is that we should do so as long as investment opportunities in less durable assets are available at the same or sufficiently close rates of return.

I suggested above that we are considering a limiting case, in which the tax policies are such that the proportion of income saved is unaffected. More realistically, the tax may fall in part on consumption, so that there may be an increase in the proportion of output going to gross saving. This will permit an increase in output over the limiting case, with an appropriate form and appropriate parameters of the production function, both during the transition and on the new equilibrium path. The nature of the problem remains the same unless, of course, the increase in saving is so great as to enable the new path to dominate the old, in terms of output, in the sense that output is higher on the new path at every point of time than on the old one. There would then be no basis for rejecting the new monetary and tax policy from the standpoint of its effect on the growth of output. There might, of course, remain grounds for so rejecting it if we set our target as the maximization of consumption rather than that of output, but this would open up a whole new set of considerations which are foreign to this paper.

There are other problems which we should bear in mind, even if we have not used this occasion to try to solve them. For one thing, we have generally assumed symmetry between assets of different longevities, in the sense that rates of return were the same for given factor ratios. Realistically this may not be so. We may imagine that with continuous functions the market will insure that the real gain, or marginal rate of return, on more roundabout methods of production, is just equal to the rate of return on less durable capital. But even if the equality exists at the margin, there is no telling that marginal productivity of capital and rates of return change similarly for different types of capital as capital-labour ratios alter. Introduction of considerations of risk would even suggest that rates of discount might be different for returns from capitals of different longevities.

What is more, we must keep it well in mind that we have been dealing with very special forms of production functions. I have my own pet peeves with some of the implications of Cobb-Douglas functions, although stating them explicitly in terms of gross output at least removes one of my *bêtes-noirs*, that of strictly positive net marginal returns to capital, which imply the possibility of infinite extensibility of the capital-labour and capital-output ratios. But this is still a rather special form of function, to be justified by convenience rather than any conviction of economic relevance. Just how our results would be altered as we move out of the entire class of constant-elasticity-of-substitution, linear and homogeneous functions might merit examination.

Indeed, I am not sure but that for many purposes we are not better off with the simple one-input production function of the earlier part of this paper than with the Cobb-Douglas, diminishing returns affair to which we turned later. As an old devotee (if for no other reason) I continue to find usefulness in Harrod-Domar type formulations. Whether because of the existence of unemployment, a pool of labour at perfectly elastic supply to be drawn upon in a non-industrial sector, the mobility of capital to draw upon labour in other areas, or a Malthusian view of the adaptability of population, there may be some considerable relevance to a production function which sees gross output as directly proportional to the stock of capital but varying inversely with the durability of that capital.

Our results would be somewhat different, it might also be noted, if we allowed for varying saving ratios as the path of output were altered in response to changes in the durability of capital. Such allowance might be called for if we applied current Friedman-Modigliani-type consumption functions.

With all, I hope that I have demonstrated that not only «rational saving-investment decisions» but also the effects of investment upon the rate of growth of output cannot be kept «independent of the durability of the structures and equipment involved». And perhaps we have the makings of an economic explanation — or justification — for the rapidly ageing young buildings I had seen in that socialist country. Or perhaps a better conclusion for socialist planners would be that as long as they are Number Two they should try harder.

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