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Observability and Constrained Optima

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DISCUSSION PAPERS

Observability and Constrained Optima

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ABSTRACT

When the asset market is incomplete, competitive equilibria are typically constrained suboptimal: there exist desirable variations in the distribution of assets which improve on the competitive allocation.

Observability of the asset and commodity demand functions of individuals suffices in order to determine desirable variations in the distribution of assets without ambiguity.

On the contrary, observability only of the asset demand functions is compatible with the claim that the competitive allocation is optimal.

INTRODUCTION

When the asset market is complete, competitive equilibria are optimal[1]: no variation in the distribution of assets can improve on the competitive allocation. Nevertheless, the optimality criterion does not employ knowledge of the economy beyond the information which can be recovered from the observable demand behavior of individuals. Observability of the individual demand functions suffices in order to determine ordinal variations in welfare.

When the asset market is incomplete, competitive equilibria are typically constrained suboptimal[2]: there exist desirable variations in the distribution of assets; variations, that is, which improve on the equilibrium allocation for every individual, after prices and quantities in the commodity spot markets adjust to maintain market clearing.

Constrained suboptimality implies that the market fails to make optimal use of the restricted set of assets for the allocation of risk[3]. It may be objected, however, that this claim ignores the possibly restricted information under which the market operates. The preference and endowment characteristics of individuals are unobservable. What is observable, at least in principle, is the demand behavior of individuals.

This paper considers whether the information which can be recovered from the observable characteristics of individuals

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1. Arrow (1951, 1953); Debreu (1951).

2. Geanakoplos and Polemarchakis (1986).

3. Hart (1975) first constructed an example of an economy with an incomplete asset market and multiple equilibria in which all individuals prefered one competitive allocation to another; he did not suggest a definition of optimality appropriate for economies with an incomplete asset market. Grossman (1979) suggested a definition which was extended by Gale (1981). Newbery and Stiglitz (1982, 1984) introduced the definition of constrained optimality which was formalized in Geanakoplos and Polemarchakis (1986).

suffices in order to improve on the competitive allocation.

According to a first proposition, if the observable characteristics of individuals consist of their demand functions for assets and commodities as asset as well as commodity prices and exogenous revenue in the asset market vary, the information which can be recovered suffices in order to determine desirable variations in the distribution of assets without ambiguity. According to a second proposition, the result fails if the observable characteristics of individuals are restricted to the demand for assets as only asset prices and exogenous revenue in the asset market vary. Indeed, it is not then possible to contradict the claim that the competitive allocation is optimal.

THE ECONOMY

Exchange occurs over two periods. The resolution of uncertainty in the second period is described by states of nature $s = 0, 1, \ldots, S$.

Commodities $\ell = 0, \ell, \ldots, L$ are traded in spot markets in the second period after the resolution of uncertainty. A commodity bundle in state s is $x_s = (\ldots, x_{s,\ell}, \ldots)$; a commodity bundle is $x = (\ldots, x_s, \ldots)$.

Assets $a = 0, 1, \ldots, A$ are traded in the first period and payoff in the second. Assets are denominated in commodity 0. The payoff of asset a in state s is $r_{s,a}$. The vector of payoffs of asset a is $r_a = (\ldots, r_{s,a}, \ldots)$, a row vector; the vector of asset payoffs in state s is $r_s = (\ldots, r_{s,a}, \ldots)$, a column vector. The matrix of asset payoffs or asset structure is $R = (\ldots, r_s, \ldots)$. A portfolio is $y = (\ldots, y_a, \ldots)$.

Individuals are $h = 0, 1, \ldots, H$. An individual, h, is characterized by his initial endowment $e^h = (\ldots, e^h_s, \ldots)$, a commodity bundle; and by his von Neumann-Morgenstern objective function[1]

$$W^h = \sum_s u^h_s .$$

defined on the consumption set X^h of non-negative consumption bundles: $x \geq 0$. The consumption set in state s is X^h_s: $x_s \geq 0$; it is the domain of the state-contingent utility index u^h_s. Evidently, the consumption set is $X^h = \prod_s X^h_s$.

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1. It would not affect the argument to suppose that the objective function takes the form

$$W^h = \sum_s \pi_s u^h ;$$

that is, with objective probabilities and a state-independent utility index. However, our argument will make essential use of the additive separability of the objective function.

We make the following assumptions:

(i) The matrix of asset payoffs, R, has full row rank.

(ii) There are at least two assets: $(A+1) \geq 2$, and two commodities: $(L+1) \geq 2$.

(iii) There exists a portfolio $\bar{y}$ such that $R'\bar{y} > 0$.

(iv) For each state, s, there exists a portfolio $\bar{y}_s$ such that $r_s' \bar{y}_s \neq 0$; equivalently $r_s \neq 0$.

(v) For each individual, h, the initial endowment is strictly positive: $e^h \varepsilon \text{ Int } X^h$, the interior of the consumption set.

(vi) For each individual, h, and every state of nature, s, the cardinal utility index u_s^h is a continuous, strictly monotonically increasing and strictly concave function which takes values on the extended real line. Everywhere on Int X_s^h, the interior of its domain of definition, u_s^h is twice continuously differentiable, $Du_s^h >> 0$, and $D^2 u_s^h$ is negative definite. Along any sequence $x_s^n \to x_s$, with $x_s^n \varepsilon \text{ Int } X_s^h$ while $x_s \varepsilon \text{ Bd } X_s^h$, the boundary of X_s^h, $x_s \neq 0$, $((x_s^n)' \, Du_s^h (x_s^n) \, / \, ||Du_s^h (x_s^n)||) \to 0$.

Assumption (i) eliminates redundant assets which do not affect the argument; (ii) allows for trade in the asset and commodity markets; (iii) is a non-trivial restriction on the asset structure; evidently, it guarantees a direction of preference over portfolios for all objective functions which are monotonically increasing consumption; (iv) guarantees that all states are accessible through the asset market; with individual objective functions separable across states, inaccessible states could be handled separately without affecting the argument. Note that assumptions (iii) and (iv) are together weaker than the alternative assumption that there exists a riskless portfolio: a portfolio $\bar{y}$ such that $R'\bar{y} >> 0$, or, after appropriate normalization of the price level at each state, $r'_s \bar{y} = 1$ for all s. Assumptions (v) and (vi) are strong but standard.

Without loss of generality, and in order to simplify notation, we suppose that the portfolio $\bar{y}$ coincides with the asset $a = 0 : r_a > 0$.

Remark: Our construction allows for consumption in the first period as a special case. It suffices to interpret consumption in state $s = 0$ as consumption in the first period and to suppose that some asset, say $a = 0$, pays off 1 at $s = 0$ and 0 at $s \neq 0$. Note that assumption (i) is then immediately satisfied.

The asset structure is complete if the matrix of asset payoffs has full column rank as well. Equivalently, if and only if $(A + 1) = (S + 1)$. If $(A + 1) < (S + 1)$, the asset structure is incomplete.

An allocation is $\underset{\sim}{x} = (\ldots, x^h, \ldots)$ such $x^h \varepsilon X^h$ for all h. The allocation of initial endowments is $\underset{\sim}{e} = (\ldots, e^h, \ldots)$.

The asset return structure is held fixed. The economy is thus a pair $(\underset{\sim}{e}, \underset{\sim}{W}) = (\ldots, e^h, \ldots, W^h, \ldots)$. The space of economies is a finite dimensional manifold sufficiently rich in perturbations. Generic sets of economies are open subsets of full lebesgue measure.

An allocation $\underset{\sim}{x}$ is feasible if and only if $\sum_h x^h \leq \sum_h e^h$.

An allocation $\underset{\sim}{x}$ fails to be optimal if and only if there exists a feasible allocation $\underset{\sim}{x}'$ such that $W^h(x'^h) \geq W^h(x^h)$ for all individuals h, with some strict inequality.

Commodity prices in state s are $p_s = (p_{s,0}, \hat{p}_s) = (p_{s,0}, p_{s,1}, \ldots, p_{s,l}, \ldots)$, a strictly positive vector; their domain is p_s. Commodity prices are $p = (\ldots, p_s, \ldots)$; their domain is $P = \prod_s p_s$.

Asset prices are $q\ (q_0, \hat{q}) = (q_0, q_1, \ldots, q_a, \ldots)$; their domain is Q, where $q \varepsilon Q$ if and only if $q = R\pi$ for the same strictly positive vector $\pi = (\ldots, \pi_s, \ldots)$.

Remark: Asset prices q do not allow for arbitrage if and only if $q' y > 0$ whenever $R'y > 0$. The domain of asset prices Q coincides with the set of asset prices which do not allow for arbitrage. Note that π is, up to normalization, the measure with respect to which asset prices satisfy the martingale property. Also, from assumption (ii), the restriction $q \in Q$ is not trivial: $-\bar{y} \notin Q$.

Asset and commodity prices (q, p) and (q', p') are equivalent up to normalization if there exist positive scalars k and $k_0, \ldots, k_S$ such that $q' = k\, q$ and $p'_s = k_s\, p_s$. Thus, by normalizing, we may set $q = 1$ and $p_{s,0} = 1$ and consider asset and commodity prices to be $\hat{q}$ and $\hat{p} = (\ldots, \hat{p}_s, \ldots)$, respectively. The domains of normalized asset and commodity prices are $\hat{Q}$ and $\hat{P}$, respectively.

Let prices be (q, p) and suppose that, in addition to his initial endowment at each state s in the second period, individual h receives in the first period exogenous revenue t^h. Individual h then expresses excess demand y^h for assets and $z^h = (\ldots, z^h_s, \ldots)$ for commodities so as to

$$\text{Max} \quad W^h(e^h + z) = \sum_s u^h_s(e^h_s + z_s)$$

$$\text{s.t.} \quad p'_s z_s = p_{s,0}\, r'_s y, \quad \text{for all } s, \tag{1}$$

$$q'y = t^h.$$

On an open neighborhood of $QxPx\{0\}$, a solution to the individual optimization problem (1) exists and is unique. The solution is characterized by the first order necessary and sufficient conditions

$$Du^h_s = \lambda^h_s\, p_s, \qquad \text{for all } s,$$

$$\sum_s \lambda_s^h \, p_{s,0} \, r_s = \mu^h q,$$

$$p_s' \, z_s^h = p_{s,0} \, r_s' \, y^h, \qquad \text{for all } s, \tag{2}$$

$$q' \, y^h = t^h,$$

where μ^h and $\lambda^h = (\ldots, \lambda_s^h, \ldots)$ are strictly positive lagrange multipliers. The excess demand function (y^h, z^h) is continuously differentiable; $z^h >> -e^h$.

For simplicity, when $t^h = 0$, we write $(y^h, z^h)(q, p, 0) = (y^h, z^h)(q, p)$ and consider the domain of the excess demand functions to be QxP.

We denote by $(\hat{y}^h, \hat{z}^h)$, where $\hat{z}^h = (\ldots, \hat{z}^h, \ldots)$, the excess demand functions for assets and commodities other than the numeraire. With $t = 0$, the domain of the function is $\hat{Q}\mathrm{x}\hat{P}$, the domain of normalized asset and commodity prices.

Aggregate excess demand as a function of asset and commodity prices is

$$(y, z) = \sum_h (y^h, z^h)$$

with domain QxP. For assets and commodities other than the numeraire, aggregate excess demand is $(\hat{y}, \hat{z})$; its domain is $\hat{Q}\mathrm{x}\hat{P}$.

A competitive equilibrium consists of asset and commodity prices (q^*, p^*) such that

$$(y, z)(q^*, p^*) = 0$$

The associated allocation is $\underset{\sim}{x}^*$.[1]

A competitive equilibrium exists.

A competitive equilibrium (q^*, p^*) is regular if

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[1]. Claims proved in Geanakoplos and Polemarchakis (1986) are stated in this section without proof.

$$|D_{(\hat{q},\ \hat{p})}\ (\hat{y},\ \hat{z})\ (\hat{q}^*,\ \hat{p}^*)| \neq 0.$$

For a fixed portfolio y^h and commodity prices p_s in state s, individual h expresses excess demand ζ_s^h so as to

$$\text{Max} \quad u_s^h\ (e_s^h + \zeta_s)$$
$$\text{s.t.} \quad p_s'\ \zeta_s = p_{s,0}\ r_s'\ y^h. \tag{3-s}$$

Suppose that $y^h = y^h\ (q,\ p)$ for some asset and commodity prices $(q,\ p)$; in particular, competitive equilibrium prices. On a neighborhood of $(y^h\ (q,\ p),\ p_s)$ a solution to the optimization problem (3-s) exists and is unique. The solution is characterized by the first order necessary and sufficient conditions.

$$Du_s^h = v_s^h\ p_s$$
$$p_s'\ \zeta_s^h = p_{s,0}\ r_s'\ y^h\ , \tag{4-s}$$

where v_s^h is a positive lagrange multiplier. The demand function ζ_s^h is continuously differentiable; $\zeta_s^h \gg e_s^h$.

The function $\zeta^h = (\ldots,\ \zeta_s^h,\ \ldots)$ is well defined and continuously differentiable on an open neighborhood of $(y^h\ (q,\ p),\ p)$.

For commodities other than the numeraire, we denote the excess demand function by $\hat{\zeta}^h\ (\ldots,\ \hat{\zeta}_s^h,\ \ldots)$; its domain is an open neighborhood of $(y^h\ (q,\ p),\ \hat{p})$.

Evidently

$$z_s^h\ (q,\ p,\ t^h) = \zeta_s^h\ (y^h\ (q,\ p,\ t^h),\ p),\ \text{for all } s;$$

also

$$\lambda_s^h(q, p, t^h) = v_s^h(y^h(q, p, t^h), p), \text{ for all } s.$$

A distribution of assets is $\underset{\sim}{y} = (y^0, \underset{\sim}{\hat{y}})$ with $y^0 = \sum_{h \neq 0} y^h$.

Variations in the distribution of assets are $d\underset{\sim}{y} = (dy^0, d\underset{\sim}{\hat{y}})$.

Let (q^*, p^*) be competitive equilibrium prices and let $\underset{\sim}{y}^* = (y^{0*}, \underset{\sim}{\hat{y}}^*)$ be the associated distribution of assets.

The aggregate excess demand function

$$\zeta = \sum_h \zeta^h$$

is well defined on a neighborhood of $(\underset{\sim}{\hat{y}}^*, p^*)$. For commodities other than the numeraire, the aggregate excess demand function is $\hat{\zeta}$ with domain an open neighborhood of $(\underset{\sim}{\hat{y}}^*, \hat{p}^*)$.

A regular competitive equilibrium is strongly regular if

$$| D_{\hat{p}} \hat{\zeta} (\hat{p}; \underset{\sim}{\hat{y}}^*) | \neq 0.$$

Remark: From the additive separability of the individual objective functions, it follows that the jacobian $D_{\hat{p}} \hat{\zeta} = \sum_h D_{\hat{p}} \hat{\zeta}^h$ is block-diagonal. Thus, the condition for strong regularity is equivalent to the condition

$$|D_{\hat{p}_s} \hat{\zeta}_s (\hat{p}_s; \underset{\sim}{\hat{y}}^*)| \neq 0, \text{ for all } s.$$

For a generic set of economies, the set of competitive equilibria is finite up to normalization, and all competitive equilibria are strongly regular.

Let (q^*, p^*) be a strongly regular competitive equilibrium, and let $d\underset{\sim}{y}$ be a variation in the distribution of assets.

The variation in spot commodity prices in state s which maintains market clearing is

$$d\hat{p}^*_s = D_{\hat{\underset{\sim}{y}}} \hat{p}^*_s \, d\hat{\underset{\sim}{y}} = -\,(D_{\hat{p}_s} \hat{\zeta}^*_s)^{-1} \, (D_{\hat{\underset{\sim}{y}}} \hat{\zeta}^*_s) \, d\hat{\underset{\sim}{y}} \;; \qquad (5\text{-}s)$$

the terms $(D_{\hat{p}_s} \hat{\zeta}^*_s)$ and $(D_{\hat{\underset{\sim}{y}}} \hat{\zeta}^*_s)$ can be computed explicitly, while the former is invertible by the assumption of strong regularity.

The variation in the utility attained by individual h in state s is

$$du^{*h}_s = -\lambda^{*h}_s \, (\hat{\zeta}^{*h}_s)' \, (D_{\hat{\underset{\sim}{y}}} \hat{p}^*_s) \, d\hat{\underset{\sim}{y}} + \lambda^{*h}_s \, p_{s,0} \, r'_s \, dy^h. \qquad (6\text{-}s)$$

Let

$$(\lambda^{*h} \times \zeta^{*h})' = (\ldots, \lambda^{*h}_s \, (\zeta^{*h}_s)', \ldots)$$

and

$$(D_{\hat{\underset{\sim}{y}}} \hat{p}^*)' = (\ldots, D_{\hat{\underset{\sim}{y}}} \hat{p}^*_s, \ldots)' .$$

Then

$$dW^{*h} = -\,(\lambda^{*h} \times \zeta^{*h})' \, (D_{\hat{\underset{\sim}{y}}} \hat{p}^*) \, d\hat{\underset{\sim}{y}} + \mu^{*h} \, q^{*\prime} \, dy^h \, . \qquad (7)$$

Let B^* be the matrix with rows $(\lambda^{*h} \times \zeta^{*h})' \, (D_{\hat{\underset{\sim}{y}}} \hat{p})$, $h = 0, \ldots, H$, and T^* the matrix with rows $(-q^{*\prime}, \ldots, q^{*\prime})$ and $(\ldots, q^{*\prime}, \ldots)$. Let $d\underset{\sim}{W}^* = (\ldots, dW^{*h}, \ldots)$; then

$$d\underset{\sim}{W}^* = (B^* + T^*) \, d\hat{\underset{\sim}{y}} \, .$$

A strongly regular competitive equilibrium is constrained suboptimal if there exists a variation in the distribution of assets dy^* such that

$$d\underset{\sim}{W}^* = (B^* + T^*) \, d\hat{\underset{\sim}{y}} > 0 \, .$$

We refer to $d\underset{\sim}{y}^*$ as a desirable variation in the distribution of assets.

If the asset market is incomplete, $(A+1) < (S+1)$, for economies in a generic set, all competitive equilibria are constrained suboptimal.[1]

A competitive equilibrium (q^*, p^*) is optimal if the associated allocation $\underset{\sim}{x}^*$ is optimal.

If the asset market is complete, $(A+1) = (S+1)$, all competitive equilibria are optimal.

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[1]. The equilibrium is strongly constrained suboptimal if $q^{*\prime} dy^{*h} = 0$ for all h; in this case, the condition for constrained suboptimality reduces to

$$d\underset{\sim}{W}^* = B^* \, d\hat{y}^* > 0 \;;$$

all competitive equilibria are strongly constrained suboptimal as long as $(A+1) \geq 3$ and $0 < 2L \leq H < SL$. To obtain strong constrained suboptimality, an upper bound on the number of individuals is necessary. This is evident by observing that the rank of the matrix B^* cannot exceed $(S+1)L$; indeed, a tighter upper bound, SL, turns out to be necessary for the argument. Mas-Colell (1987) has shown that in an economy with a large number of diverse individuals competitive equilibrium allocations are constrained optimal.

Consider an equilibrium which is constrained suboptimal. If the characteristics of each individual, his initial endowment and his objective function are known, a desirable variation in the distribution of assets can be determined without ambiguity. Such characteristics are, however, unobservable. What is observable, in principle at least, are the demand functions of each individual for assets and possibly commodities as well. The two propositions which follow consider whether the information which can be recovered from the observable characteristics of individuals suffices in order to improve on a constrained suboptimal competitive equilibrium allocation.

We restrict our attention to strongly regular equilibria.

Proposition 1: Consider a constrained suboptimal equilibrium (q^*, p^*) for the economy $(\tilde{e}, \tilde{W})$ is a generic set. Observability of the individual demand functions for assets and commodities (y^h, z^h), as asset and commodity prices and exogenous revenue (q, p, t^h) vary, suffices in order to determine unambiguously desirable variations in the distribution of assets.

Proposition 2: Consider a competitive equilibrium (q^*, p^*) for the economy $(\tilde{e}, \tilde{W})$; let $\tilde{x}^*$ be the associated allocation and y^* the associated distribution of assets. There exists an economy $(\tilde{e}, W')$ such that

(i) (q^*, p^*) is a competitive equilibrium for the economy $(\tilde{e}, \tilde{W}')$, with the same associated allocation $\tilde{x}^*$, and the same associated distribution of assets $\tilde{y}^*$.

(ii) For all h,

$$D_q\, y'^h\,(q^*, p^*, 0) = D_q\, y^h\,(q^*, p^*, 0)$$

and $$D_{t^h}\, y'^h\,(q^*, p^*, 0) = D_{t^h}\, y^h\,(q^*, p^*, 0),$$

where y'^h is the demand function for assets of the individual (e^h, W'^h).

(iii) The allocation $\underset{\sim}{x}^*$ is optimal for the economy $(\underset{\sim}{e}, W')$.

The two propositions differ in what they take to be the observable characteristics of individuals. Proposition 1 allows variations in asset as well commodity demands as asset prices, revenue as well as spot commodity prices vary to be observable. Proposition 2 does not allow for variations in spot commodity prices. In the former case the information recovered suffices in order to determine desirable variations in the distribution of assets. In the latter case this is not so: the information recovered is compatible with the claim that the competitive equilibrium is optimal and hence no desirable variation in the distribution of assets exists.

Proof of proposition 1: We break down the proof into a sequence of steps.

Step 1: By hypotheses, the demand function (y^h, z^h) is observable; that is, the asset and commodity demands of individuals can be observed as asset and commodity prices, as well as the exogenous revenue of the individual in the first period, (q, p, t^h), vary. It follows that the functions ζ_s^h are observable as well. In particular, the derivatives $D_{\hat{p}_s} \zeta_s^h$ and $D_{y^h} \hat{\zeta}_s^h$ at the competitive equilibrium commodity prices, p^*, and portfolio y^{*h} are observable. It follows from (7) by substituting from (5-s) and (6-s) that to determine $d\underset{\sim}{y}^*$ it suffices to determine the lagrange multipliers λ_s^{*h}. Note that without loss of generality we may set $\mu^{*h} = 1$.

Step 2: Consider the individual optimization problem (3-s) in the second period, state s, for a given portfolio y^h. Normalizing prices, i.e. setting $p_{s,0} = 1$ and $dp_{s,0} = 0$, and totally differentiating the first order conditions (4-s) we obtain

$$\begin{bmatrix} d\zeta_s^h \\ d\lambda_s^h \end{bmatrix} = \begin{bmatrix} S_s^h & -v_s^h \\ -v_s^{h'} & -e_s^h \end{bmatrix} \begin{bmatrix} \lambda_s^h & dp_s & \\ \zeta_s^{h'} & dp_s - r_s' & dy^h \end{bmatrix}$$

where

$$\begin{bmatrix} S_s^h & -v_s^h \\ -v_s^{h'} & -e_s^h \end{bmatrix} = \begin{bmatrix} D^2 u_s^h & -p_s \\ -p_s' & 0 \end{bmatrix}^{-1} \tag{8-s}$$

It follows that

$$\begin{aligned} D_{\hat{p}_s} \hat{\zeta}_s^h &= \lambda_s^h \hat{S}_s^h - \hat{v}_s^h \hat{\zeta}_s^{h'} , \\ D_{y^h} \hat{\zeta}_s^h &= \hat{v}_s^h r_s' , \\ D_{\hat{p}_s} \lambda_s^h &= - \lambda^h \hat{v}_s^{h'} + e_s^h \hat{\zeta}_s^{h'} , \\ D_y^h \lambda_s^h &= e_s^h r_s' , \end{aligned} \tag{9-s}$$

where "^" denotes the projection to the subspace of commodities other than the numeraire.

Step 3: From the individual optimization problem (3-s) we obtain the indirect utility function $u_s^h\ (p_s, y)$.

Let

$$W^h\ (p, y) = \sum_s u_s^h\ (p_s, y)$$

and consider the individual optimization problem

$$\begin{aligned} &\text{Max} \quad W^h\ (p, y) \\ &\text{s.t.} \quad q'\ y = t^h. \end{aligned} \tag{10}$$

A solution to the optimization problem (10) exists and is unique. The solution $y^h\ (q, p)$ coincides with the asset demand function of the individual obtained from the optimization problem, (1); it is characterized by the first order necessary and sufficient conditions

$$\sum_s r_s \lambda_s^h = q$$

$$q' y = t^h \tag{11}$$

where λ_s^h are the lagrange multipliers associated with the optimization problem (3-s), while by normalizing the objective function we have set the langrange multiplier associated with the budget constraint in the asset market equal to 1. Totally differentiating the first order conditions (11) and setting

$$\begin{bmatrix} S_0^h & -v_0^h \\ -v_0^{h'} & -e_0^h \end{bmatrix} = \begin{bmatrix} \sum_s e_s^h r_s r_s' & -q \\ -q' & 0 \end{bmatrix}^{-1} \tag{12}$$

which is well defined since $e_s^h < 0$ for all s while the collection of vectors $\{\ldots, r_s, \ldots\}$ has rank (A+1), we obtain that

$$D_{p_s} y^h = S_0^h r_s \{\lambda_s^h v_s^{h'} + e_s^h \zeta_s^{h'}\} ,$$

$$D_q y^h = S_0^h - v_0^h y^{h'}, \tag{13}$$

$$D_{t^h} y^h = v_0^h .$$

The system of equations (13) evaluated at (q^*, p^*) suffices in order to determine unambiguously the multipliers λ_s^{*h} and hence to complete the argument. Since the asset demand function y^h is observable, its derivatives are observable. The derivative $D_{t^h} y^h$ is equal to v_0^{*h}. From $D_q y^h$ we can then compute S_0^{*h}. Finally from $D_q\, y^h$ we can compute unambiguously λ_s^{*h} (and e_s^{*h}) as long as v_s^{*h} and ζ_s^{*h} are not colinear; but this is so for economies in a generic set. This completes the argument by noting that v_s^{*h} is observable from $D_{y^h}\, \zeta_s^h$ is (9-s) as long as $r_s \neq 0$, which we have assumed to be the case. QED

<u>Remark</u>: It follows from the proof of proposition 1 that observability of the individual demand function for assets and commodities suffices in order to recover the individual objective function, up to ordinal transformations. This depends essentially on the possibility of trade in the commodity spot markets; that is, it fails in the economies with only one commodity which were considered in earlier literature on recoverability[1].

Proof of proposition 2: We break down the proof into a sequence of steps.

Step 1: Suppose there exist constants $k^0, \ldots, k^h, \ldots, k^H$ such that $\lambda_s^{*h} = k^h \lambda_s^{*0}$ for all states of nature s and all individuals h; evidently $k^0 = 1$. Let $p^{**} = (\ldots, p_s^{**}, \ldots)$, where $p_s^{**} = \lambda_s^0\, p_s^*$. It follows that $DW^{*h} = k^h\, p^{**}$ for all h. From the concavity of the objective functions W^h it follows that the allocation $\tilde{x}^*$ is optimal. Hence, to complete the argument, it suffices to show that there exist objective functions W'^h for all h such that the asset and commodity demand functions (y'^h, z'^h) satisfy

(i) $(y'^h, z'^h)(q^*, p^*) = (y^h, z^h)(q^*, p^*)$;

(ii) $D_{t^h}\, y'^h (q^*, p^*, 0) = D_{t^h}\, y^h (q^*, p^*, 0)$,

and

$$D_q\, y'^h (q^*, p^*, 0) = D_q\, y'^h (q^*, p^*, 0) .$$

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1. Dybvig and Polemarchakis (1981) and Green, Lau and Polemarchakis (1979).

Also that

(iii) $\lambda_s'^{*h} = k^h \lambda_s'^{*0}$ for some scalar k^h, for all s and all h.

Step 2: For each individual, h, consider the objective function

$$W'^h = \sum_s f_s^h \circ u_s^h ,$$

where f_s^h, for each s, is a twice continuously differentiable, strictly monotonically increasing and strictly concave transformation:

$Df_s^h > 0$, $D^2 f_s^h < 0$. Evidently, $D(f_s^h \circ u_s^h) = (Df_s)(Du_s^h)$. We shall specify the transformations f_s^h for each s in order to satisfy (i), (ii) and (iii), in particular, we shall specify Df_s^{*h} and $D^2 f_s^{*h}$, the first and second derivatives at the equilibrium. Let $Df_s^{*h} = k^h \lambda_s^{*0} / \lambda_s^{*h}$ for some positive scalar k^h and for all s. From the first order conditions (2) it follows that asset and commodity demands at (q^*, p^*) remain unchanged, hence (i) is satisfied; (iii) is satisfied by construction. Concerning (ii), it follows from the expressions in (13) and (12) that the derivatives $D_q y^h$ and $D_{t^h} y^h$ at $(q^*, p^*, 0)$ remain unchanged as long as, for all s, the transformation f_s leaves e_s^{*h} unchanged: $e_s'^{*h} = e_s^{*h}$. Noting that $D^2 (f_s^h \circ u_s^h) = (D^2 f_s^h) Du_s^h (Du_s^h)' + (Df_s^h) D^2 u_s^h$ and substituting in (8-s), where e_s^h is defined, we obtain that $e_s'^{*h} = e_s^{*h}$ as long as $(\lambda_s^{*h})^2 D^2 f_s^{*h} + Df_s^{*h} - 1 = 0$. This completes the argument by observing that $D^2 f_s^{*h} < 0$ for all s as long k^h is sufficiently large. QED

Remark: Proposition 2 considers the asset demand functions to be only infinitesimally observable at the equilibrium. It is straightforward to extend the argument and allow the entire asset demand functions, as asset prices and revenue in the asset market, (q, t^h), vary, to be observable.

CONCLUSION

The criterion of optimality appropriate to a particular organization of the market should take into account the constraints under which the market operates. When the asset market is incomplete, constrained optimality does indeed restrict attention to the available assets and does not allow for instruments which the market does not have at its disposal. It ignores, however, informational constraints: With a restricted set of assets, the observable demand behavior of individuals need not reveal the underlying unobservable characteristics of individuals - their preferences and endowments. In this paper, under different assumptions on the observable behavior of individuals, we have examined whether the information which is revealed suffices in order to improve on constrained suboptimal competitive allocations.

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